Then $a-A$=water in the cask.

$A+b$=quantity of wine in cask before the $(m+1)$th draught since b gallons of wine are added.

$A+b-[(A/n)+(b/n)]+b=A\left(\frac{n-1}{n}\right)+b\left(\frac{n-1}{n}\right)$=quantity of wine before the $(m+2)$th draught.

$$A\left(\frac{n-1}{n}\right)+b\left(\frac{2n-1}{n}\right)-A\left(\frac{n-1}{n^2}\right)-b\left(\frac{2n-1}{n^2}\right)+b=A\left(\frac{n-1}{n}\right)^2-b\left(\frac{3n^2-3n+1}{n^2}\right)$$

=quantity of wine before the $(m+3)$th draught.

$$\therefore A\left(\frac{n-1}{n}\right)^p+b\left(pn^{p-1}-\frac{p(p-1)}{1.2}n^{p-2}+\ldots\ldots\right)+b$$

$$=A\left(\frac{n-1}{n}\right)^p+b\left(\frac{n^p-(n-1)^p}{n^p}\right)$$

=quantity of wine left after $(m+p)$th draught$=a\left(\frac{n-1}{n}\right)^{m+p}+b\left(\frac{n^p-(n-1)^p}{n^p}\right)$

In the present case, $a=10$, $b=1$, $1/m=\frac{1}{10}$, $m=5$, and $p=5$. Hence, substituting, we have $10\left[\frac{10-1}{10}\right]^{10}+1.\left[\frac{10^5-(10-1)^5}{10^5}\right]=7.581884401$ gallons, the quantity of wine left after putting in the last gallon of wine, and, therefore, 2.418115599 gallons=quantity of water in the cask.

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

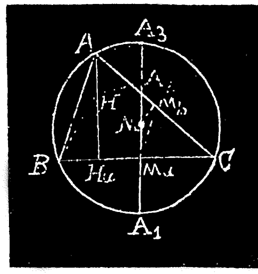
SOLUTIONS OF PROBLEMS.

71. Proposed by ROBERT J. ALEY, A. M., Ph. D., Professor of Mathematics, Indiana University, Bloomington, Indiana.

Prove by pure geometry: A perpendicular at the middle point, M_a, of the side BC of the triangle ABC meets the circumcircle in A'. On this perpendicular A'' and A''' are taken so that $M_aA''=M_aA'$ and $A''A'''=AH$. (H is the orthocenter of triangle ABC.) Prove that A''' is on the circumicircle.

Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, New Jersey, and the PROPOSER.

Let $M_aA_1=M_aA_2$, $A_2A_3=AH$, to prove A_3 on the circumference of the circle. Since A_2A_3 is a line through M, the center of the circle, the proposition is in effect to prove A_3 one extremity of the diameter through M_a.



By the conditions $AH=A_2A_3$, and is parallel to it, therefore AHA_3A_2 is a parallelogram.

Also triangles BHA and M_aMM_b are similar, hence since $2M_aM_b=AB$, we have $AH=2MM_a$.

Therefore, $A_1A_3=A_2A_3+A_2M_a+M_aA_1$

$=AH+2M_aA_1$

$=2M_aM+2M_aA_1$

$=2(MA_1)=2r$, hence A_3 is extremity of diameter.

Q. E. D.

Also solved by *CHAS. C. CROSS*, and *J. W. SCROGGS*.
Mr. Cross furnished two different solutions.

72. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

If a line with its extremities upon two curves move in any manner whatever, (the line may vary in length), and P a point upon the line which divides it in the ratio $m:n$ describe a curve, the area of this curve will be given by the formula—

$$A=\frac{(m^2+nm)A_1+(n^2+mn)A_2-mnA_3}{(m+n)^2}.$$

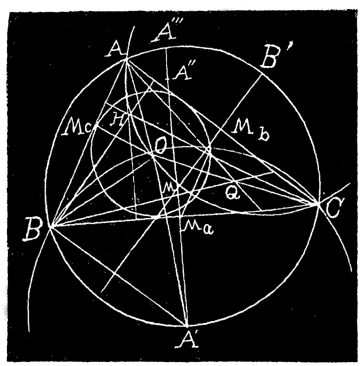
No solution of this problem has been received.

73. Proposed by ROBERT J. ALEY, A. M., Ph. D., Professor of Mathematics, Indiana University, Bloomington, Indiana.

Prove by pure geometry: (1) A', B', and C' are the middle points of the arcs BC, CA, and AB respectively. With these points as centers, circles are described passing through B and C, C and A, and A and B respectively. Prove that these circles intersect in O, the center of the incircle of the triangle ABC; (2) that O, the center of the incircle, is Nagel's point of the triangle formed by joining the middle points of the sides.

Solution by CHARLES C. CROSS, Laytonsville, Maryland, and the PROPOSER.

(1) AO cuts the circumcircle at A', for AO bisects angle A and also its subtending arc. $\measuredangle OBA'=\frac{1}{2}(A+B)$.



$\measuredangle BOA'=\frac{1}{2}(A+B)$ for it is exterior angle to triangle BOA.

$\therefore$ triangle $A'BO$ is isosceles.

$A'B=A'O$. By similar reasoning it is proved that $B'A=B'O$ and $C'A=C'O$.

$\therefore$ The circles intersect in O.

(2) It is a well known property of Nagel's point that AQ and OM_a, BQ and OM_b, CQ and OM_c are respectively parallel.